

# Z-domain Analysis of Ghost Cancellation for Television Signals

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## Abstract

This paper proposes an approach in ghost cancellation for television signals by z-domain analysis. Z-domain analysis estimates the zeros that were added by the channel. First the z-domain characteristics for different multipath channels are examined. Post-ghosts give zeros inside the unit circle, while pre-ghosts result in zeros outside the unit circle. From this the implications that the z-domain characteristics of the channel have on ghost cancellations are outlined. The zeros inside the unit circle may be cancelled directly by placing the poles of an IIR filter in the same locations. The zeros outside the unit circle resulting from pre-ghosts must be corrected by using a FIR filter. Several techniques of estimating the channel characteristics from the z-domain characteristics are described. The channel estimation can be done without any special reference signal transmitted for this purpose. However the estimates are more reliable with a known available signal, such as horizontal synchronisation signal. A blind channel characterisation method based on matching zeros is proposed. However this new approach is quite sensitive to both noise and rounding errors. The z-domain analysis performed here can also be applied to other multipath propagation problems.

## 1. Introduction

At the frequencies used for television broadcast, radio signals travel in straight lines. In addition to the direct path between the transmitting antenna and the receiving antenna, the same signal also can reach the receiver via reflections off nearby buildings and other objects. Under such conditions, the receiver will show multiple images – a strong, main image and weaker shifted images from the echoes. These multiple images are called ‘ghosts’ because they have the same information but offset in time, giving the appearance of a ghost. Ghosts severely distract television viewing. The propagation paths to each receiver have different characteristics so the only practical solution to

ghost reduction on the final display is to process the signal at the receiver. Ghost reduction can be achieved by characterizing the propagation channel for that viewer. Although the channel characteristics usually vary slowly with time, they can be treated as time invariant from frame to frame. Hence an adaptive digital filter can be used for the equalisation with the filter parameters changed dynamically according to the variations of the propagation channel.

Ghosts may also appear in cable television either through impedance mismatches, or as a result of different modes of propagation within the cable. While the causes may be different, the effect on the viewed image is similar.

## 2. Channel characterisation techniques

Several techniques have been developed by researchers to mitigate ghosting in multipath television signals.

Komiya [1] proposed an adaptive ghost reduction system consisting of a cascaded ghost reduction filter and ghost reproduction filter. The system functions as a blind deconvolution system and to satisfy the reciprocity of transfer functions of these two filters, adaptive process force the system output waveforms to coincide with the input waveforms.

Kouam et al [2] developed a method capable of equalising long delays by a method of filtering in the frequency domain using small FFTs implemented by means of a decimation technique. This technique makes use of temporal windowing, frequency sub-sampling, and automatic selection of the number of filters required. With this technique it is possible to correct echoes even at high sampling frequencies.

Greenberg [3] describes ghost cancellation using a standard ghost cancellation reference (GCR) signal. The GCR signal consists of a frequency sweep or chirp of constant amplitude. This maximises the energy within the signal while providing a flat frequency spectrum within the bandwidth of interest (DC to 5 MHz for PAL and 4.2 MHz for

NTSC). The chirp also has the property that its autocorrelation function is a narrow sinc function, enabling the channel impulse response to be obtained by correlation with a reference signal at the receiver.

There are 3 world standards [6] for the GCR signal, the specifications for which are laid down in the ITU-R recommendations [5]. Television broadcast services use a standard GCR signal, transmitted during the vertical blanking interval. A television receiver equipped with a GCR decoder uses the reference signal to characterise the channel and implement a compensation filter.

Wang et al [4] propose a frequency domain optimisation technique where a frequency division algorithm has been modified. This approach has several draw backs such as a large number of FFT and inverse FFT operations are required to calculate the filters, and the results with frequency division is not reliable under high noise and large ghost conditions.

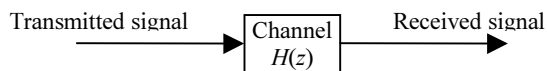
### 3. Analysis in the z-domain

This section considers the effects of the channel (including ghosts) in the z-domain. First the z-domain characteristics of the channel are investigated. Second, the implications these have on ghost cancellation are discussed. Finally, several techniques of estimating the channel characteristics from the z-domain are proposed.

While the details given here relate directly to ghosting in television signals, much of the analysis is also applicable to other multipath propagation problems.

#### 3.1 Channel characteristics

A z-domain analysis implicitly assumes that the channel is a linear time invariant (LTI) system. While this is not strictly correct, the channel characteristics change sufficiently slowly that a linear analysis provides a useful approximation. The channel effectively behaves as a filter, as shown in figure 1.

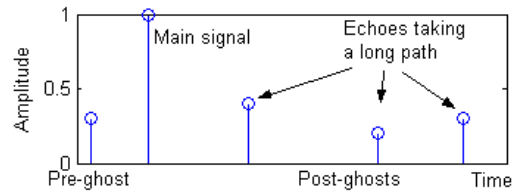


**Figure 1:** The channel as a filter.

The effect of multipath echoes is to give the channel an impulse response of the form shown in figure 2.

We define the main signal to be the strongest signal component. This is usually the direct signal, although in fringe areas, especially with irregular

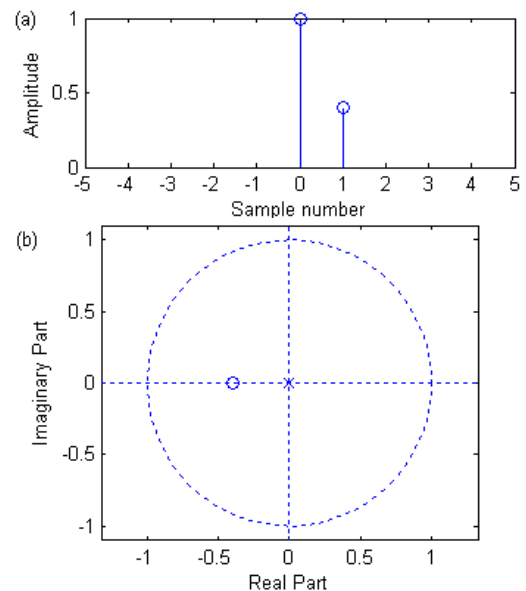
terrain, one of the reflected signals may be much stronger than the direct signal. Each additional path to the receiver, resulting from reflections off buildings or other objects, contributes an extra impulse in the impulse response. These ghosts have lower amplitude than the main signal because of losses associated with the reflection and extra propagation distance. In some circumstances (for example in cable television or when the main signal is not the direct signal) there may be one or more pre-ghosts.



**Figure 2:** Impulse response of a multipath channel.

The channel therefore can be considered as a finite impulse response (FIR) filter because of its finite extent, and isolated impulses. It is necessary to understand the characteristics of this filter in order to develop a compensation filter.

First, let us consider a single ghost with amplitude of 0.4 relative to the main signal and delayed by one sample, as depicted in figure 3a. In the z-domain (figure 3b) the ghost results in a single zero at  $z = -0.4$ .



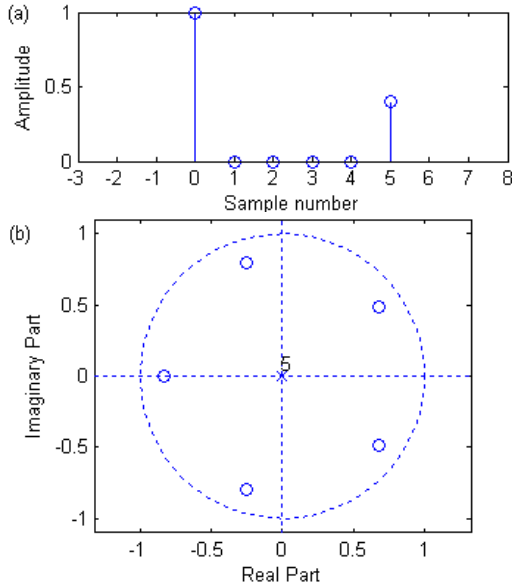
**Figure 3.** (a) Impulse response of a channel with a single post-ghost with amplitude of 0.4. (b) The pole-zero plot for this channel.

In general, the position of the zero will depend on the amplitude of the ghost. If the channel is

$$H(z) = 1 + az^{-1} \quad (1)$$

Then the zero will be located at  $z = -a$ .

This is an over-simplification. There will usually be a significant time delay between the main signal and any ghost, so the ghost will seldom occur in the subsequent sample. To illustrate the effect of such delay, figure 4 shows the same ghost delayed by 5 samples.



**Figure 4.** (a) A channel with a single post-ghost delayed by 5 samples. (b) The corresponding pole-zero plot.

As the channel is now a 5<sup>th</sup> order system:

$$H(z) = 1 + az^{-5} \quad (2)$$

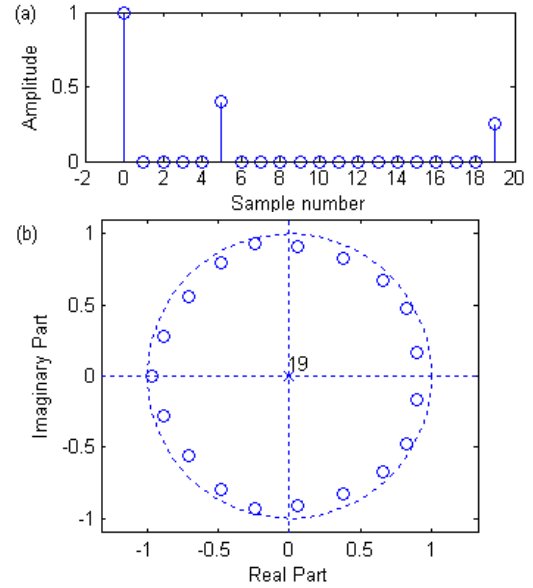
There will be 5 zeros in the z-domain. Their locations are at

$$z = (-a)^{1/5} \quad (3)$$

This has the zeros spaced at equal angles and at the same distance from the origin. For a given ghost amplitude, the more the ghost is delayed, the more zeros are introduced, and the closer they will be to the unit circle. Similarly, the larger the amplitude of the ghost, the closer the zeros will be to the unit circle.

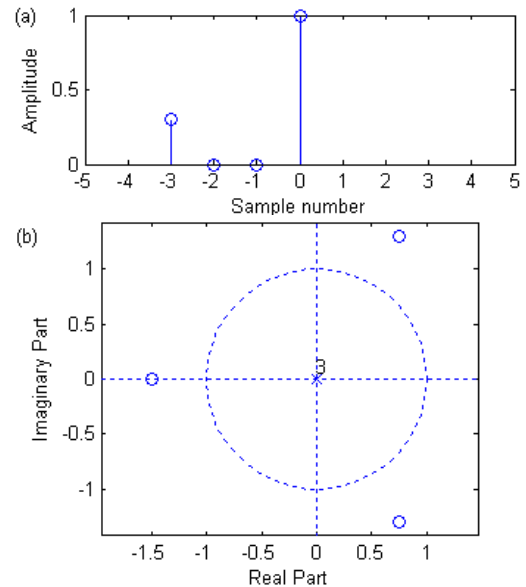
When the channel has multiple post-ghosts, the number of zeros in the system depends on the longest delay. This is illustrated in figure 5.

With multiple ghosts, the zeros are not all at the same radius or equally spaced. If one or more of the ghosts are significantly large then one or more of the zeros may move outside the unit circle. However, for post-ghosts encountered in practise, the ghost amplitude will decrease significantly with increased delay, and all of the zeros are typically inside the unit circle [3].



**Figure 5.** (a) A channel with ghosts delayed by 5 and 19 samples. (b) A channel with ghosts delayed by 5 and 19 samples.

As mentioned earlier, in some circumstances a ghost may appear before the main signal rather than after it. The effect of this is shown in figure 6.

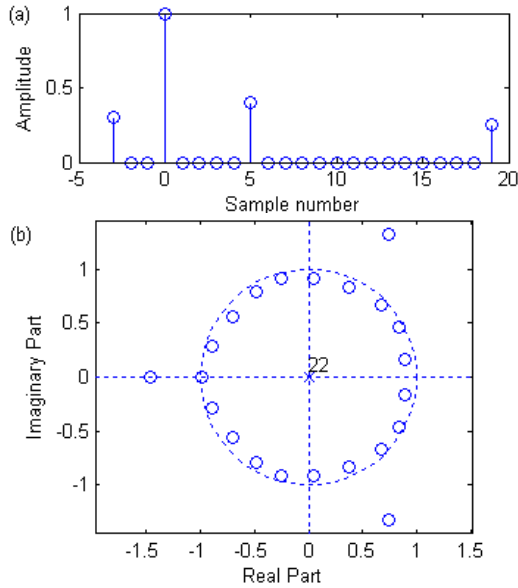


**Figure 6.** (a) A single pre-ghost channel of 0.3 at -3. (b) The corresponding pole-zero plot.

The effect of ghosts before the main signal is to place the zeros outside the unit circle. This is because the signal is increasing in amplitude with time rather than decreasing. Again, the number of zeros depends on the position of the pre-ghost, and the radius depends both on the position and amplitude.

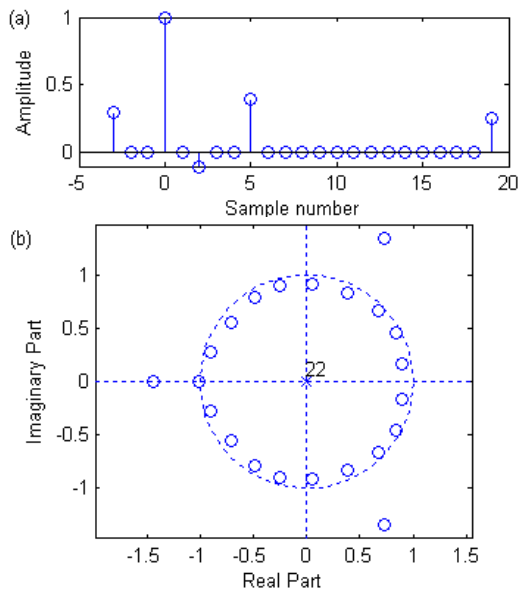
When pre-ghosts and post-ghosts are combined, the zeros outside the unit circle generally come from the pre-ghost(s) and those inside come from the post-ghosts. This is clearly seen in the example in

figure 7. Note that it is not just simple combining of the zeros of the pre- and post-ghost. All of the zeros will be perturbed by the addition of another ghost (either before or after the main signal).



**Figure 7.** (a) A Multipath channel. (b) All zeros from the post-ghosts are inside the unit circle, and those from the pre-ghosts are outside.

While the number of zeros outside the unit circle is determined primarily by the position of the pre-ghost, it is possible to have additional zeros outside the unit circle from near post-ghosts, particularly if the post-ghosts have significant amplitude to bring one or more zeros close to the unit circle. This is shown in figure 8 where an additional post-ghost results in moving one of the zeros outside of the unit circle.

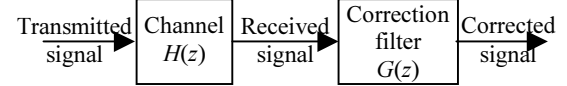


**Figure 8.** (a) An additional post-ghost with amplitude -0.1 at sample 2. (b) This post-ghost moves an additional zero to outside unit circle.

### 3.2 Implications for corrections

Ghost amplitudes in a typical channel are below -6 dB relative to the main signal. Pre-ghosts may occur up to 4  $\mu$ s before the main signal, with post-ghost up to 40  $\mu$ s after the main signal. When the video signal is sampled at 13.5 MHz, ghosts may be found between samples -54 and +540.

Ghost correction may be accomplished by a correction filter as shown in figure 9. Ideally we require  $G(z)H(z)=1$  to completely eliminate the effect of multipath reflections within the channel.



**Figure 9.** Channel correction using a filter.

Ideally, when only post-ghosts are present, all the zeros are within the unit circle. It is then possible to equalise the channel by placing the poles of  $G(z)$  on zeros of  $H(z)$  to cancel them, thus realising the inverse filter of the channel response. Since all of the poles of the inverse filter are inside the unit circle, this will result in a stable infinite impulse response (IIR) filter.

If the channel response has ghosts with amplitude  $g_i$  located at sample  $n_i$ , the channel can be represented as

$$H(z) = 1 + \sum_i g_i z^{-n_i} \quad (4)$$

The correction filter is then given as

$$G(z) = \frac{1}{H(z)} = \frac{1}{1 + \sum_i g_i z^{-n_i}} \quad (5)$$

Therefore the filter coefficients are given directly from the sampled impulse response. If the ghost amplitudes are sufficiently low that all of the zeros are within the unit circle (which is usually the case) the IIR filter may be implemented safely using its direct form. This does not even require factorising the channel response to locate the zeros.

When the channel introduces pre-ghosts, the same cancellation technique can not be used directly. Since a pre-ghost introduces zeros outside the unit circle, the resulting cancellation filter would have poles outside the unit circle, and would therefore be unstable.

Poles outside the unit circle correspond to a stable infinite anti-causal sequence, which is unrealisable because it is not causal. However, the sequence can be truncated (making it finite) and then delayed, making it causal. This process is illustrated in figure 10 for the pre-ghost shown in figure 6. When both pre- and post-ghosts are present, the correction

filter  $G(z)$  can be split into two components: a FIR filter to cancel the pre-ghost, and an IIR filter to cancel the post-ghosts. If there are no near post-ghost, the FIR filter can be calculated using only the pre-ghosts. This is then convolved with the channel impulse response to remove the pre-ghost from that response, and the resultant signal used to calculate the IIR filter.

When there are near post-ghosts, the process will need to be more complicated because the convolution will result in further impulses before the main signal coming from the near post-ghost. One way around this is to find the roots of  $H(z)$  and separate the zeros inside the unit circle from those outside. These can then be used independently for developing the corresponding FIR and IIR compensation filters. This approach is not particularly practical because finding the roots of a higher order polynomial is not trivial, and is very susceptible to round-off errors while performing the calculations.

An alternate approach is to take the pre-ghost and near post-ghosts (within 4  $\mu\text{s}$ ) and develop the FIR filter for cancelling the pre-ghost from this. While this still involves factorising a polynomial, this is more practical. Once the FIR filter has been determined, it can be convolved with the complete signal as before to give the IIR component. Rather than performing factorisation, the FIR filter may also be developed iteratively, for example using adaptive filter algorithms [3].

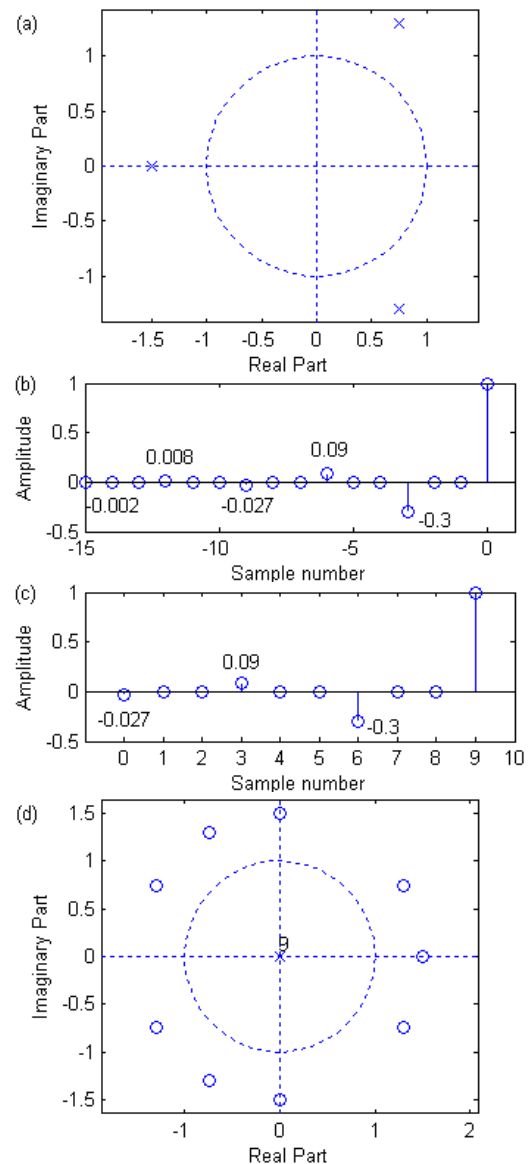
If the channel is such that the post-ghosts are sufficiently large that they result in zeros on or outside the unit circle, cancellation is generally not possible. Zeros on the unit circle will result in loss of information at the corresponding frequency that cannot be recovered. While a zero outside the unit circle may be compensated for, this is generally not practical. Placing a pole just outside the unit circle will require a very slowly decaying anti-causal sequence. Consequently, a large number of coefficients must be used for the generated FIR filter to represent the sequence with any accuracy, making the filter too large for practical implementation.

### 3.3 Characterising the channel

The analysis in the preceding section assumes that the channel impulse response (or equivalently the locations of the zeros in the  $z$ -domain) are known. This may be determined in number of ways, either directly or indirectly.

The direct approach is to periodically transmit an impulse signal, for example during the vertical blanking interval, and measuring the impulse

response directly. While this gives the impulse response directly, it is limited by how much energy can be placed within a single impulse.



**Figure 10.** Development of a compensation filter for a pre-ghost. (a) Desired pole locations. (b) Corresponding (stable) infinite anti-causal sequence. (c) Delayed truncated sequence. (d) Its corresponding pole-zero plot.

This approach may be extended by transmitting other known signals, and deconvolving the response to calculate the impulse response. Best results can be obtained when the input signal has components over the full range of frequencies. Rather than use dedicated signal, the line synchronisation pulse may be used. While not ideal – the rectangular pulse has a sinc frequency content with nulls – it does have significantly higher energy than a single impulse.

If the channel response to the horizontal synchronisation pulse is considered in the  $z$ -domain, there are two components:

$$Y(z) = H(z)X(z) \quad (6)$$

The horizontal synchronisation pulse is a finite length signal, so  $X(z)$  will consist only of zeros (all the poles are at the origin). As shown in section 2.1,  $H(z)$  also consists only of zeros. Since  $Y(z)$  is the product of the two, it will directly combine the zeros of the two components. Since  $X(z)$  is a known signal, with zeros sitting on the unit circle, these zeros may be eliminated from  $Y(z)$  leaving only those from  $H(z)$ . In the case of the horizontal synchronisation signal, this is straight forward as the zeros of  $X(z)$  all lie on the unit circle.

From a  $z$ -domain analysis, if the zeros of the signal are well separated from those of the channel, the zero locations may be found with more confidence. As all of the post-ghost zeros are close to, and inside the unit circle, if the zeros of the transmitted signal are outside the unit circle, this permits easy identification of the zeros associated with the channel. One signal with such characteristics is a truncated exponentially growing sequence. Another, simpler, signal is a linear ramp.

There is not such problem with pre-ghost zeros since the pre-ghost is relatively close to the main signal and sufficiently low in amplitude that the zeros outside the unit circle can be easily identified.

This approach may be extended to achieve blind channel characterisation. When an arbitrary input signal is applied to the channel, again the zeros of the signal will combine with the zeros of the channel. This time the zeros are not able to be separated because the input signal is unknown. However, with multiple independent input signals, the zeros associated with the channel should remain in the same locations, whereas the zeros from the input signals will change. The advantage of this approach is that it requires no additional information - the broadband television signal itself can be used to characterise the channel.

While this approach to blind channel characterisation works in theory, it is subject to three important limitations. First, for most wide band television signals, the  $z$ -transform of a single line of video will tend to have zeros spread around the unit circle. This is exactly where channel zeros are also likely to be found. Second, locating the zeros of the  $z$ -transform of a line of video is not a straight forward because of the high system order. There are large number of zeros spread around the unit circle, and their precise location requires high precision calculations. Round-off errors resulting

from limited precision arithmetic can significantly perturb the zeros. Third, the higher the order of the system, the more sensitive the zero locations are to any noise that will inevitably be present on the received signal.

In practice, these limitations mean that this blind channel characterisation approach really only works with short signals under relatively low noise conditions.

#### 4. Conclusions:

$Z$ -domain analysis can be used to characterise the propagation channels for broadcast television. It is shown that a channel may be considered as a FIR system. For channels with only post-ghosts it is possible to devise an equalisation filter that completely corrects for the effects of the channel provided that the zeros from the channel are all within the unit circle. When pre-ghosts are present, an exact stable equalisation filter is not possible. However, an arbitrarily close approximation may be derived using a combination of FIR and IIR filters. We have also shown that it is possible to use the locations of zeros to estimate the channel characteristics without sending known reference signals specifically transmitted for ghost cancellation.

#### 5. References

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